Enumeration

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where $\{x_1, \ldots, x_k\} = \mathrm{free}(\varphi)$

Query evaluation

Consider a query language L (e.g., FO, CQ)

Let $\varphi(x_1, \ldots, x_k)$ be a query formulated in L.
Let D be a db.

Task:

Evaluate φ on D, i.e., compute the query result

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Special case $\mathsf{free}(\varphi) = \emptyset$ (i.e., $k = 0$): Boolean queries:
Evaluate φ on D means Decide if $D \models \varphi$

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CAVEAT: These notions & results do not handle updates of the db!

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Goal: preprocessing time & delay (& update time) as small as possible

Constant delay enumeration after linear time preprocessing

- Let φ be a fixed query
- Let C be a class of databases
- Goal:
  Find an algorithm which, upon input of a db D,
  uses preprocessing time (pseudo-)linear in $\|D\|$ and then
  enumerates $\varphi(D)$ with delay $O(1)$

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- Question:
  For which φ, C is this possible?

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Overview

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Bounded degree databases

Graph $G = (V, E)$:

$$\begin{array}{rcl} \text{degree of a node } v & : & \text{the number of neighbours of } v \text{ in } G \\ \text{degree of } G & : & \max\{\text{degree}(v) : v \in V\} \end{array}$$

Database D:

$$\text{degree of } D \quad : \quad \text{degree of the Gaifman graph of } D$$

Gaifman graph of D:

the graph $G = (V, E)$ with $V = \text{adom}(D)$ and an edge between two distinct nodes $a, b \in V$ iff some tuple in some relation of D contains a and b

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$$f(\varphi, d) \quad = \quad 2^{d^{2^{O(\|\varphi\|)}}} \quad = \quad 3\text{-exp}(\|\varphi\| + \lg\lg d)$$

and the 3-fold exponential blow-up is unavoidable assuming FPT $\neq$ AW[$*$].

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Generalisation to the dynamic setting (update time $f(\varphi, d)$) and FO+MOD [Berkholz, Keppeler, S. 2017] and FOC($\mathbb{P}$) [Kuske, S. 2017]

FO+MOD queries and FOC($\mathbb{P}$) queries

Movie	
Name	Actor
Alien	Sigourney Weaver
Blade Runner	Harrison Ford
Blade Runner	Sean Young
Brazil	Jonathan Pryce
Brazil	Kim Greist
Casablanca	Humphrey Bogart
Casablanca	Ingrid Bergmann
Gravity	Sandra Bullock
Gravity	George Clooney
Resident Evil	Milla Jovovich
Terminator	Arnold Schwarzenegger
Terminator	Linda Hamilton
Terminator	Michael Biehn
⋮	⋮

Is the number of movies with Ingrid Bergmann even?

In FO+MOD:

$\exists^{0 \bmod 2} y \;\; \mathit{Movie}(y, \text{"Ingrid Bergmann"})$

FO+MOD = extension of first-order logic with modulo-counting quantifiers $\exists^{i \bmod m} y \; \psi(y, \bar{z})$

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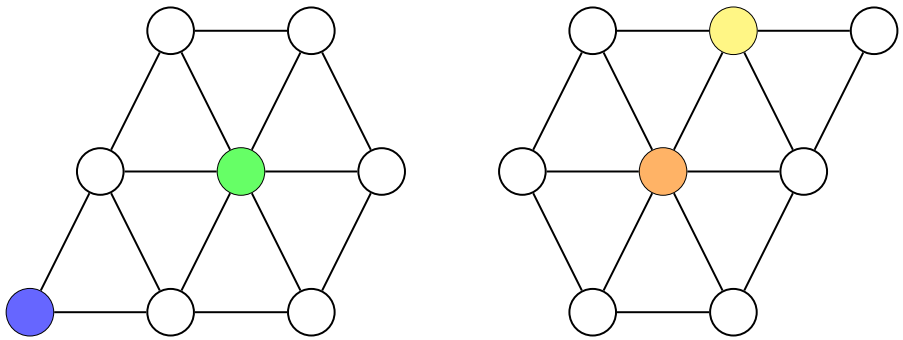
FOC($\mathbb{P}$) = extension of first-order logic with formulas of the form $P(t_1, \ldots, t_r)$ for $P \in \mathbb{P}$ of arity r, and where each t_i is a counting term built using integers, $+$, $\cdot$, and basic counting terms $t(\overline{x})$ of the form $\#\overline{y}.\psi(\overline{x}, \overline{y})$

Hanf normal form for FO+MOD

- A type τ with k centres and radius r:



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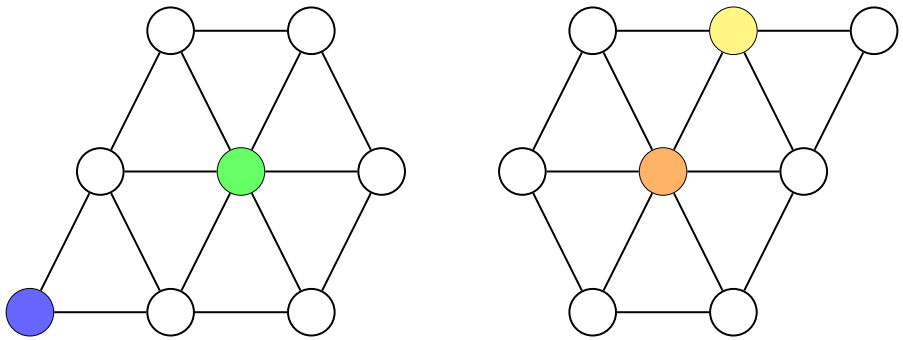

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- $\mathcal{N}_r^D(\bar{b})$ is the induced substructure of D on

$$N_r^D(\bar{b}) \quad = \quad N_r^D(b_1) \cup \cdots \cup N_r^D(b_k)$$

where

$$N_r^D(b_i) \quad = \quad \{a \in \mathrm{adom}(D) \ : \ \mathrm{dist}^D(b_i, a) \leqslant r\}$$

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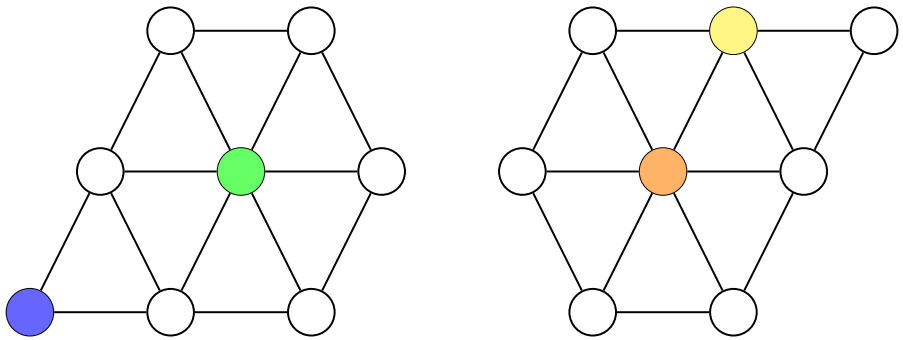

Example type with $k = 4$ centres and radius $r = 1$

- $\mathcal{N}_r^D(\bar{b})$ is the induced substructure of D on

$$N_r^D(\bar{b}) \quad = \quad N_r^D(b_1) \cup \cdots \cup N_r^D(b_k)$$

where

$$N_r^D(b_i) \quad = \quad \{a \in \mathrm{adom}(D) \ : \ \mathrm{dist}^D(b_i, a) \leqslant r\}$$

- Sphere-formula $\mathrm{sph}_\tau(\bar{x})$:

$$(D, \bar{a}) \models \mathrm{sph}_\tau(\bar{x}) \quad \Longleftrightarrow \quad (\mathcal{N}_r^D(\bar{a}), \bar{a}) \cong \tau$$

Hanf normal form for FO+MOD

A Hanf normal form $\psi(\overline{x})$ is a Boolean combination of

- sphere-formulas $\mathrm{sph}_\rho(\overline{x})$ and
- Hanf-sentences $\exists^{\geqslant m} y\ \mathrm{sph}_\tau(y)$ and $\exists^{i \bmod m} y\ \mathrm{sph}_\tau(y)$ where τ is a type with 1 centre and radius r.

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Theorem (Heimberg, Kuske, S., 2016)

There is an algorithm which receives as input a degree bound $d \geqslant 2$ and a FO+MOD*-formula $\varphi(\overline{x})$, and constructs a d-equivalent formula $\psi(\overline{x})$ in Hanf normal form.*
The algorithm's runtime is $f(\varphi, d) = 3\text{-}exp(\|\varphi\| + \lg\lg d)$.

Boolean queries under updates $f(\varphi, d) = 3\text{-exp}(\|\varphi\| + \lg\lg d)$

Observation: There is a dynamic algorithm that receives as input

- a degree bound $d \geqslant 2$,
- a Boolean FO+MOD-query φ, and
- a db D of degree $\leqslant d$,

and computes

- within $f(\varphi, d)\|D\|$ preprocessing time a data structure
- that can be updated in time $f(\varphi, d)$

and allows to return the query result $\varphi(D)$ with answer time $O(1)$.

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Proof Idea:

1. Transform φ into Hanf normal form ψ.
2. For each type τ occurring in ψ, store the number A_τ of elements in adom(D) whose r-neighbourhood has type τ.
3. Inserting/deleting a tuple in D only affects the type of a fixed number of elements in adom(D) $\rightsquigarrow$ update affected A_τ-values.

Enumeration under updates $f(\varphi, d) = 3\text{-exp}(\|\varphi\| + \lg\lg d)$

Theorem (Berkholz, Keppeler, S. 2017): There is a dynamic algorithm that receives as input

- a degree bound $d \geqslant 2$,
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- within $f(\varphi, d)\|D\|$ preprocessing time a data structure
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Proof Idea:

Proof idea: Reduction to coloured graphs

Input:

Database D

FO+MOD-query $\varphi(x_1, ..., x_k)$

Same approach as in [Durand, S., Segoufin 2014],

but now we have to take care of updates!

Proof idea: Reduction to coloured graphs

Input:

Database D

FO+MOD-query $\varphi(x_1, ..., x_k)$

$\sigma_k := \{E, C_1, ..., C_k\}$

σ_k-structure $\mathcal{G}$

$\overline{v} \in \psi_k(\mathcal{G})$

$$\psi_k(x_1, \ldots, x_k) := \bigwedge_{i=1}^{k} C_i(x_i) \wedge \bigwedge_{i \neq j} \neg E(x_i, x_j)$$

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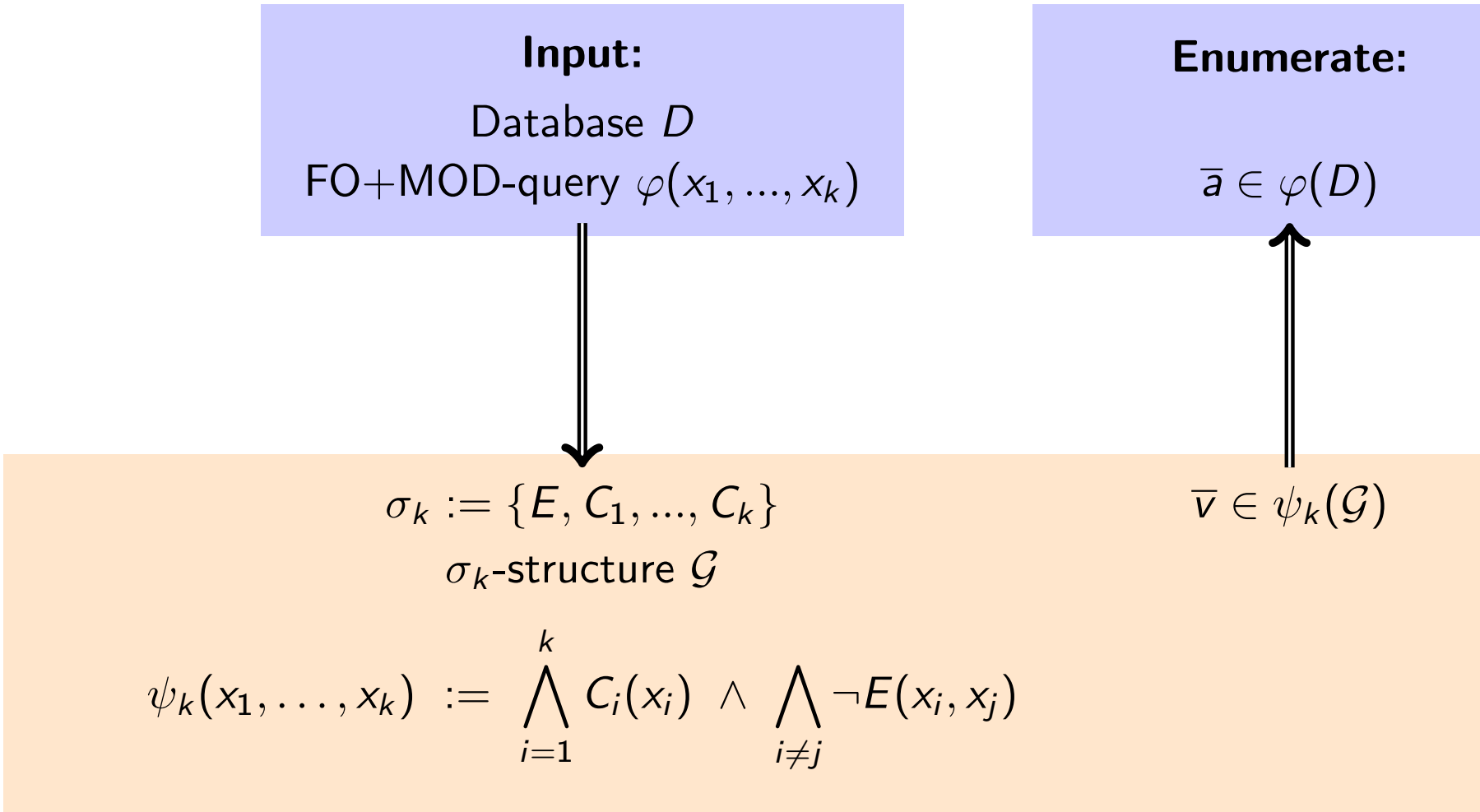

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Representing Databases by Coloured Graphs

$$\varphi(x_1, \ldots, x_k) \quad \equiv_d \quad \bigvee_{i \in \mathcal{I}} \mathrm{sph}_{\tau_i}(x_1, \ldots, x_k) \quad \& \text{ sentences}$$

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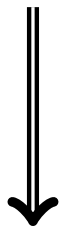

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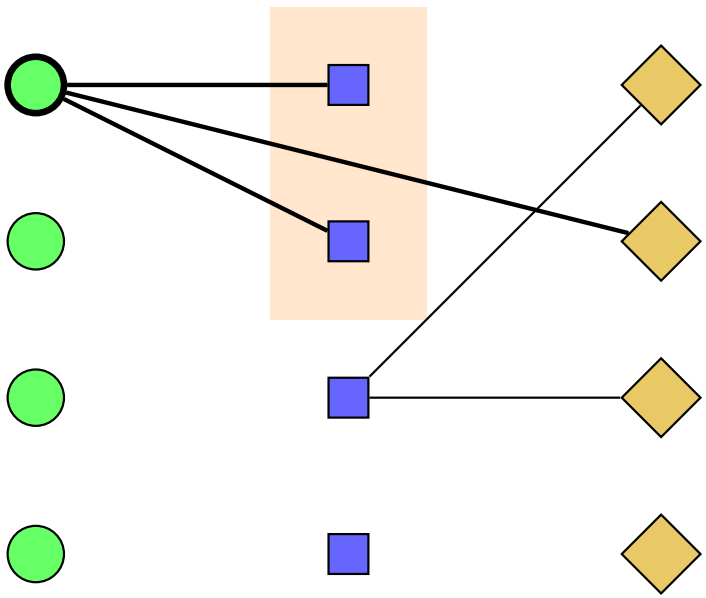
When a tuple is inserted/deleted in D, update the coloured graph $\mathcal{G}$.

Enumeration of $\psi_k(\mathcal{G})$ with delay $O(k^3 d)$

$$\psi_k(x_1, \ldots, x_k) \quad := \quad \bigwedge_{i=1}^{k} C_i(x_i) \ \wedge \ \bigwedge_{i \neq j} \neg E(x_i, x_j)$$



for all $u_1 \in C_1^{\mathcal{G}}$ **do**
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Output EOE.

function $\textsc{Enum}(u_1, \ldots, u_i)$
    if $i = k$ **then**
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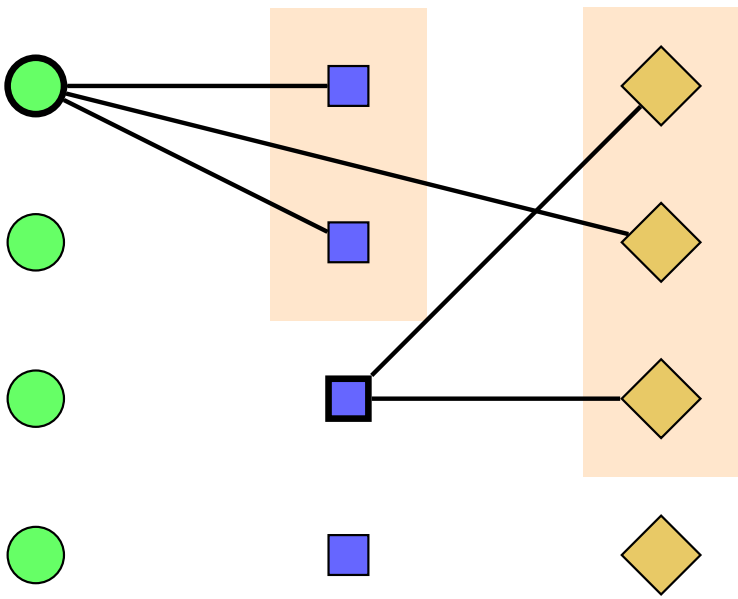

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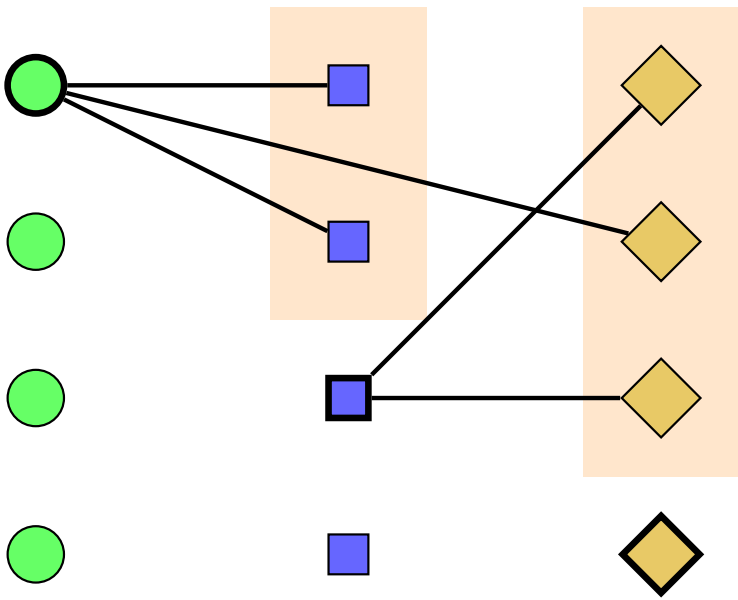

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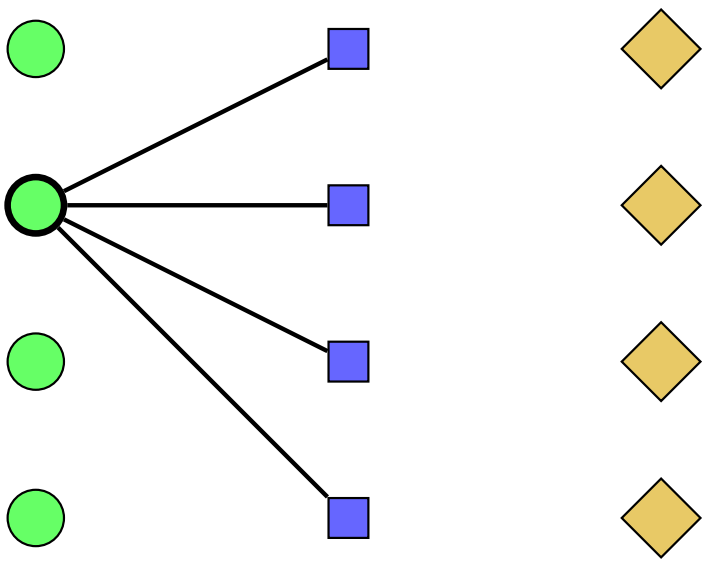

Problem: Too few blue nodes

for all $u_1 \in C_1^{\mathcal{G}}$ **do**
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Output EOE.

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Handling small colours

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The set $\mathcal{S}$ can be computed in time $O((dk)^k)$.

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$$\overline{s} \in \mathcal{S} \quad \Longleftrightarrow \quad \text{ex. } \overline{a} \text{ such that } \; (\overline{s}, \overline{a}) \in \varphi(D)$$

The enumeration procedure

1: **for all** $(u_1, \ldots, u_s) \in \mathcal{S}$ **do**
2: $\quad$ $\texttt{Enum}(u_1, \ldots, u_s)$.
3: Output the end-of-enumeration message $\texttt{EOE}$.
4:
5: **function** $\textsc{Enum}(u_1, \ldots, u_i)$
6: $\quad$ **if** $i = k$ **then**
7: $\quad\quad$ output the tuple $(u_1, \ldots, u_i)$
8: $\quad$ **else**
9: $\quad\quad$ **for all** $u_{i+1} \in C_{i+1}^{\mathcal{G}}$ **do**
10: $\quad\quad\quad$ **if** $u_{i+1} \notin \bigcup_{j=1}^{i} N^{\mathcal{G}}(u_j)$ **then**
11: $\quad\quad\quad\quad$ $\texttt{Enum}(u_1, \ldots, u_i, u_{i+1})$

where $N^{\mathcal{G}}(u_j) := \{v \in V^{\mathcal{G}} : (u_j, v) \in E^{\mathcal{G}}\}$.

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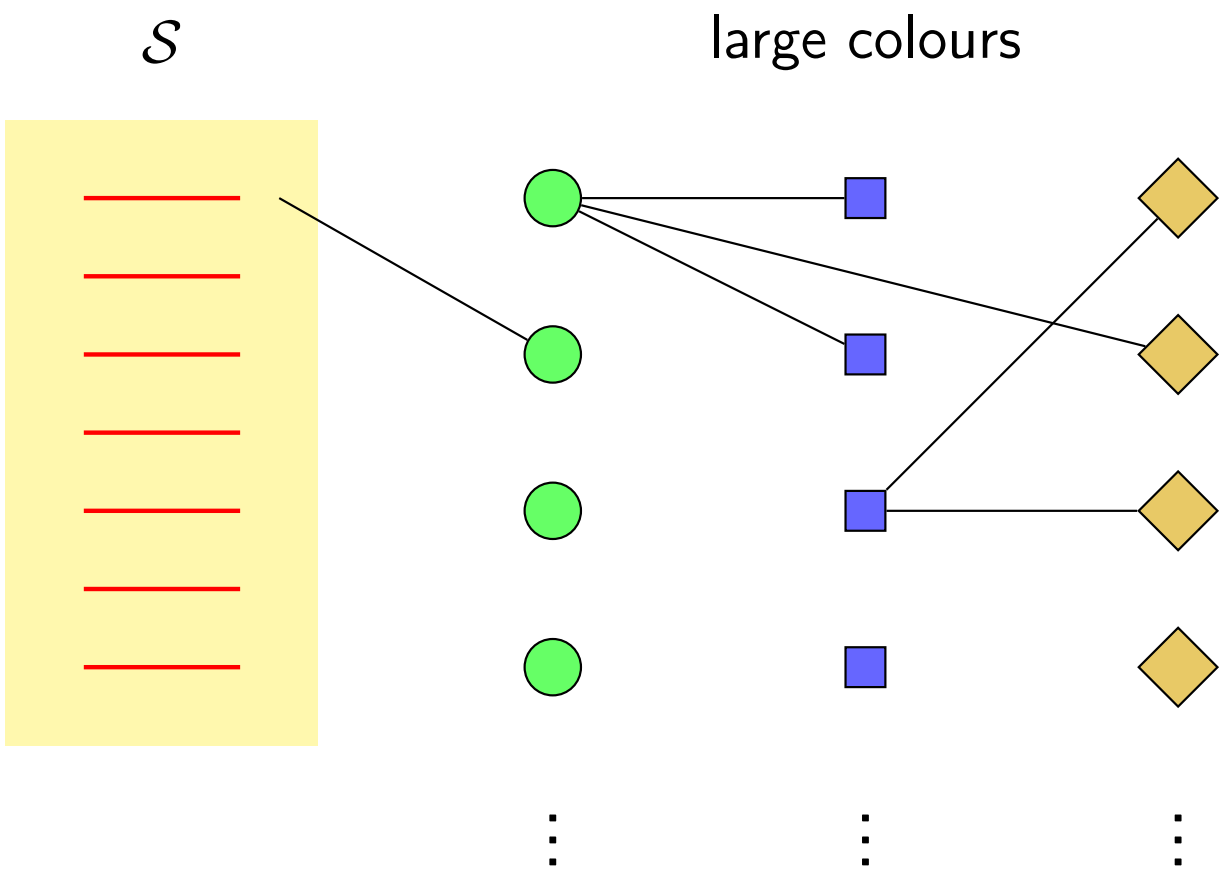

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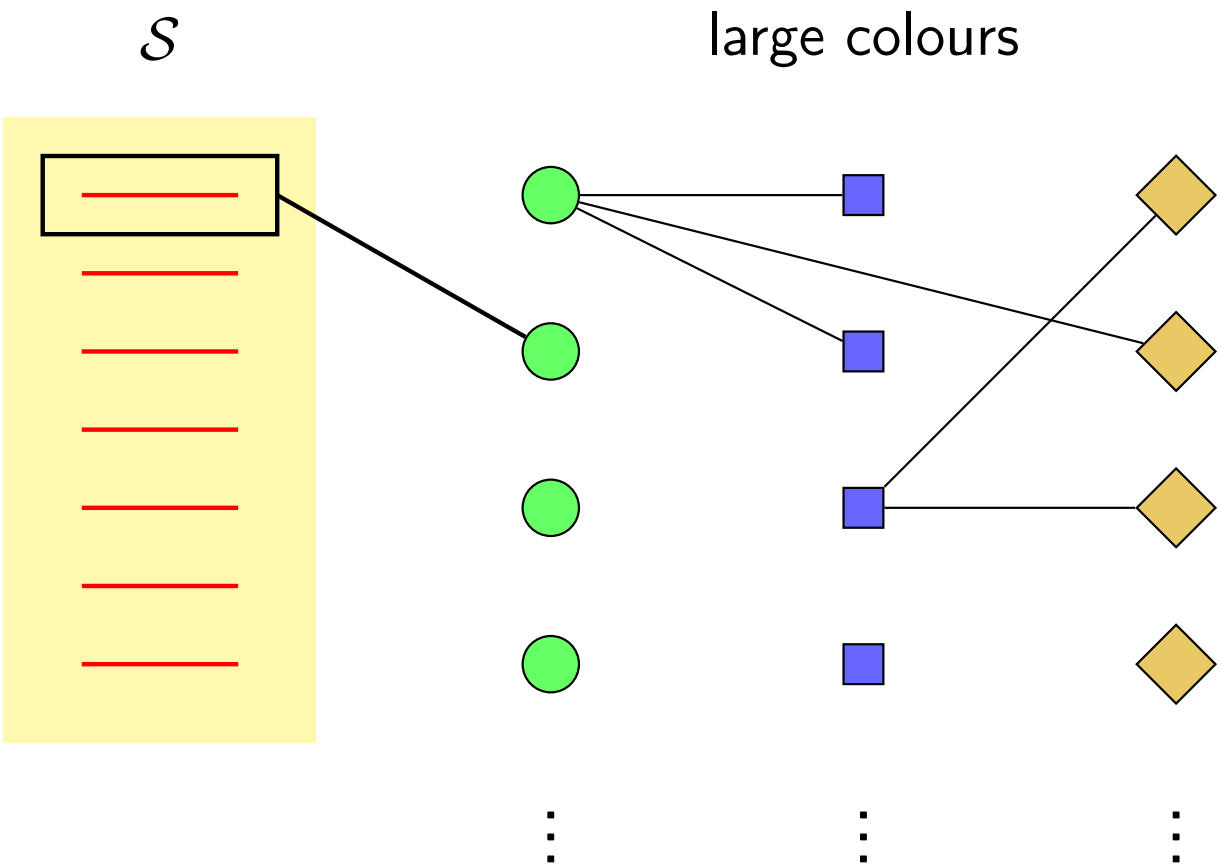

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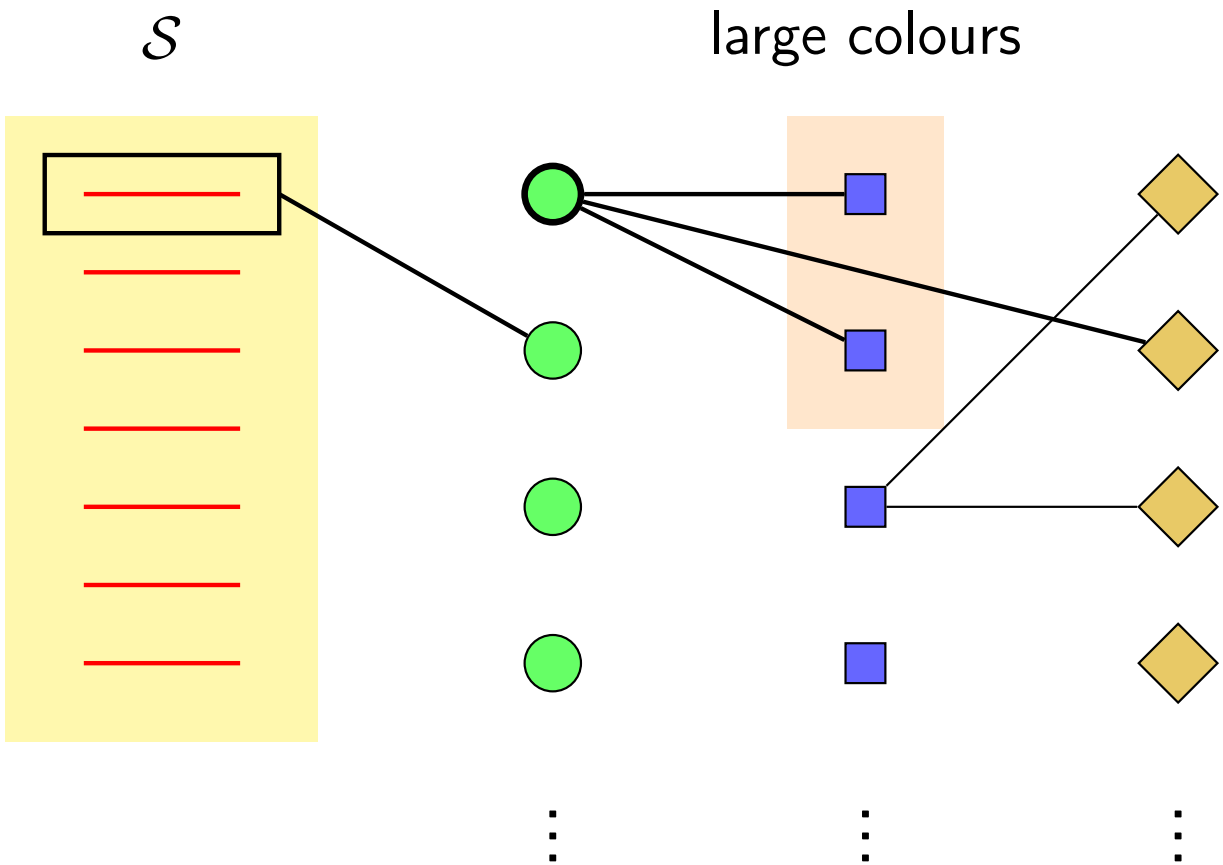

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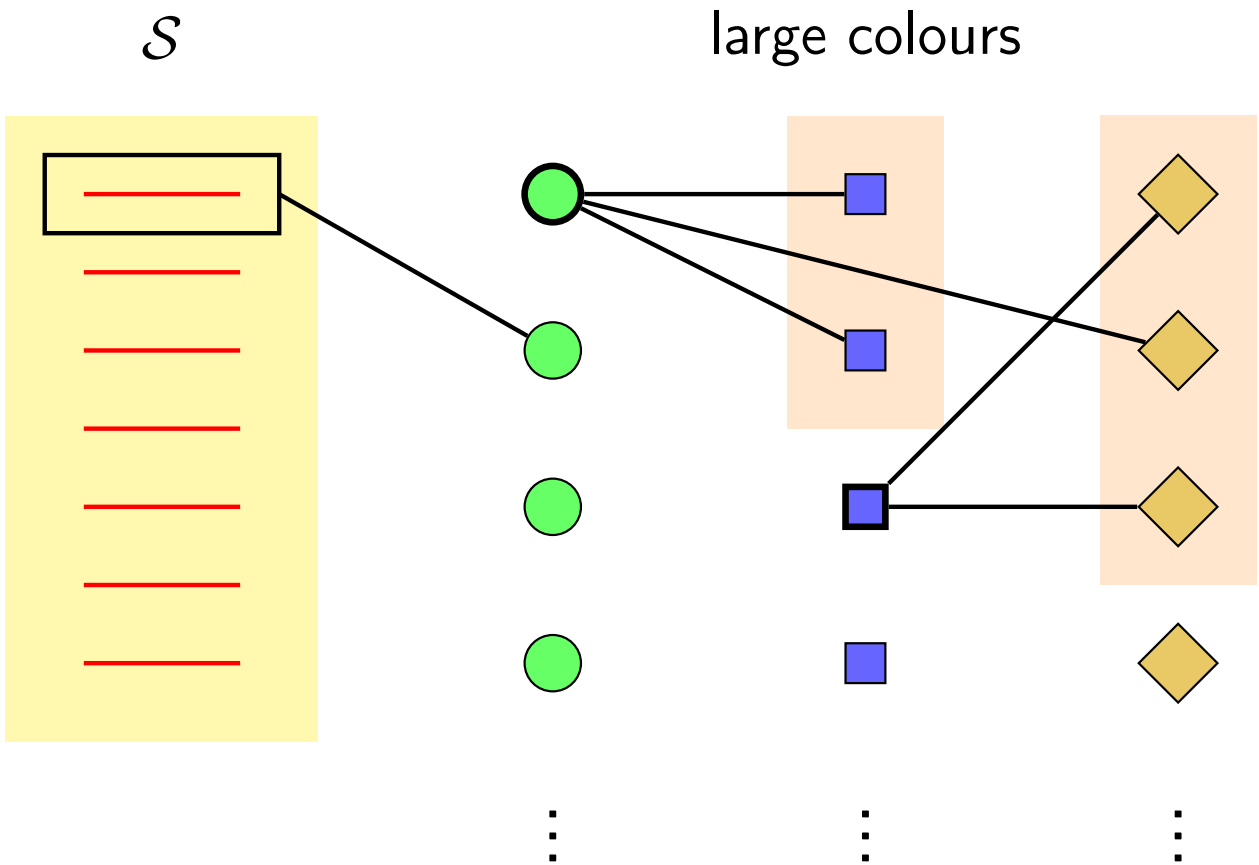

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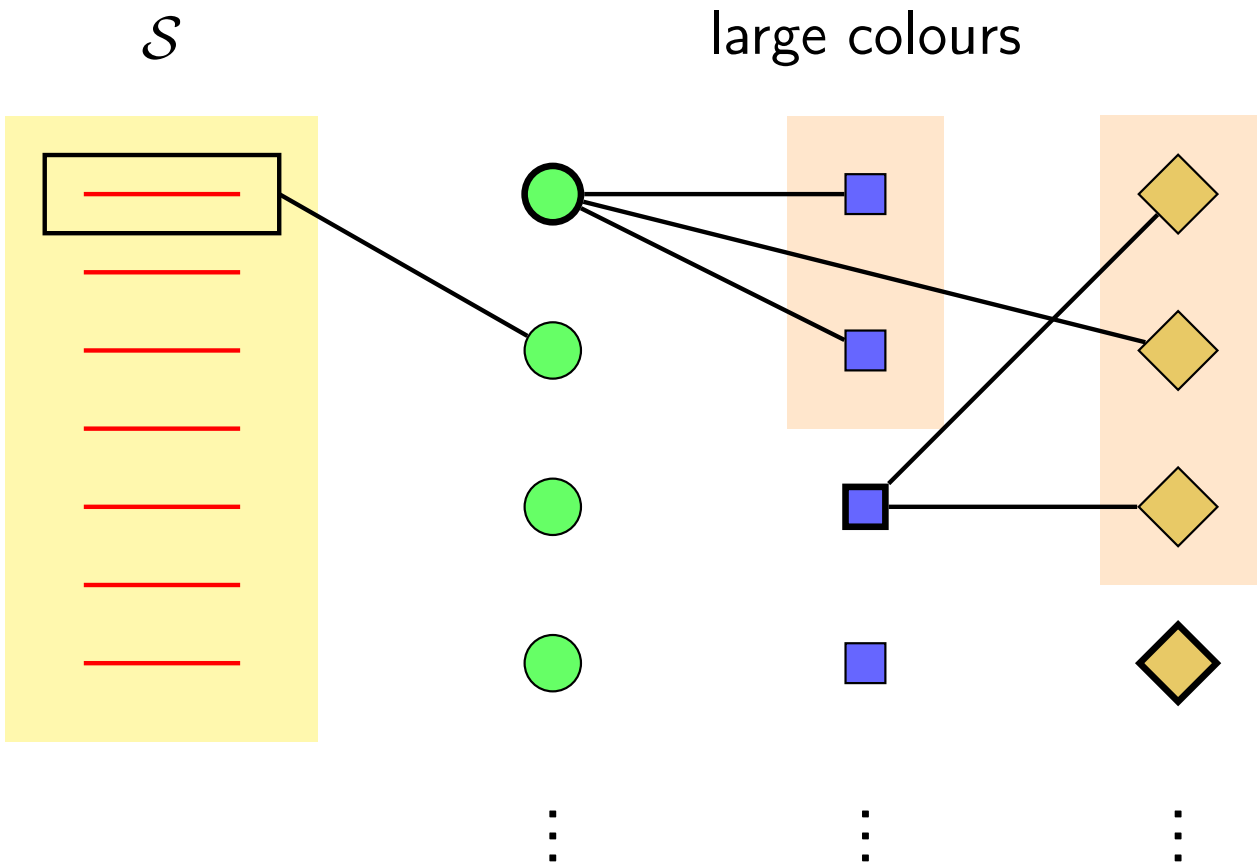

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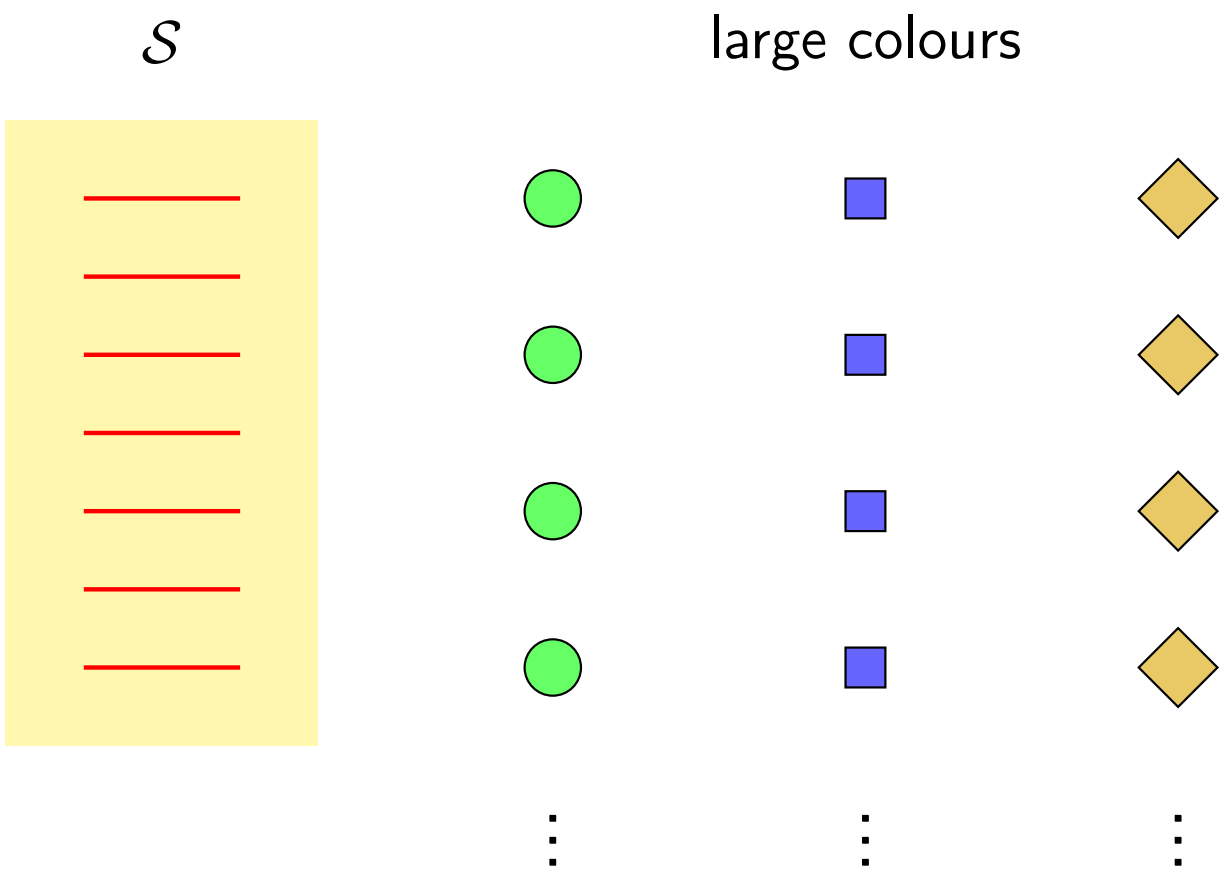

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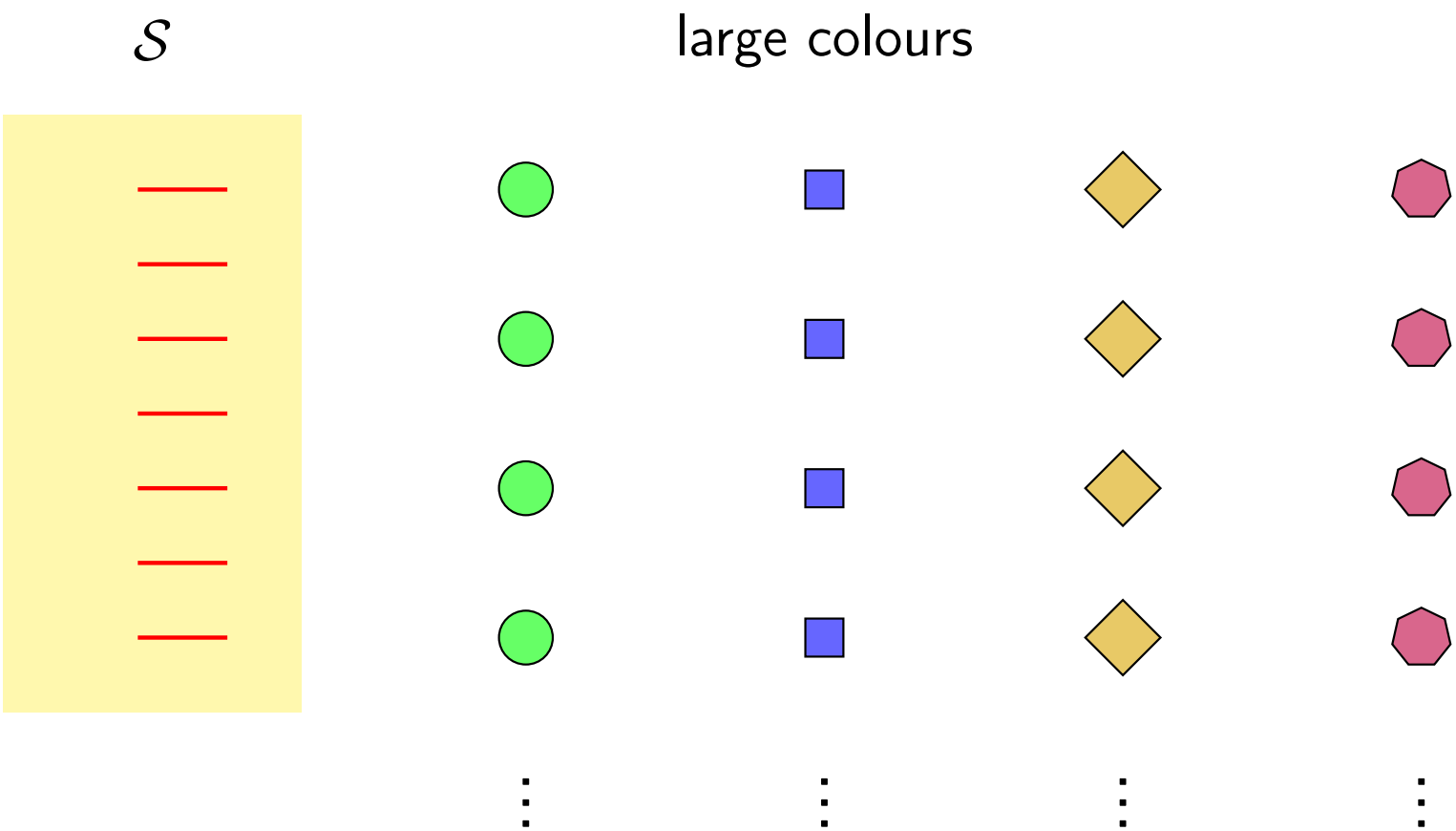

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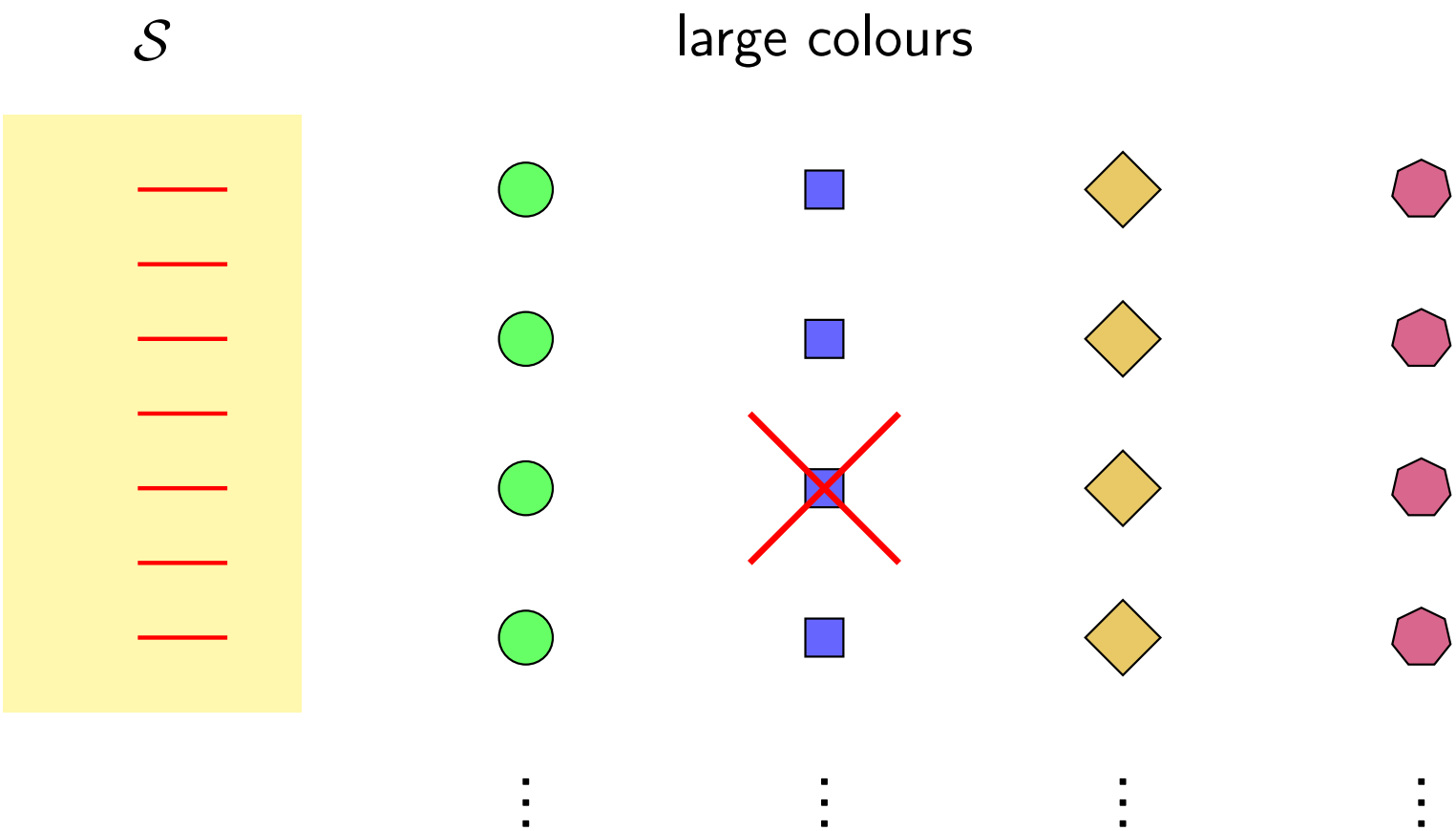

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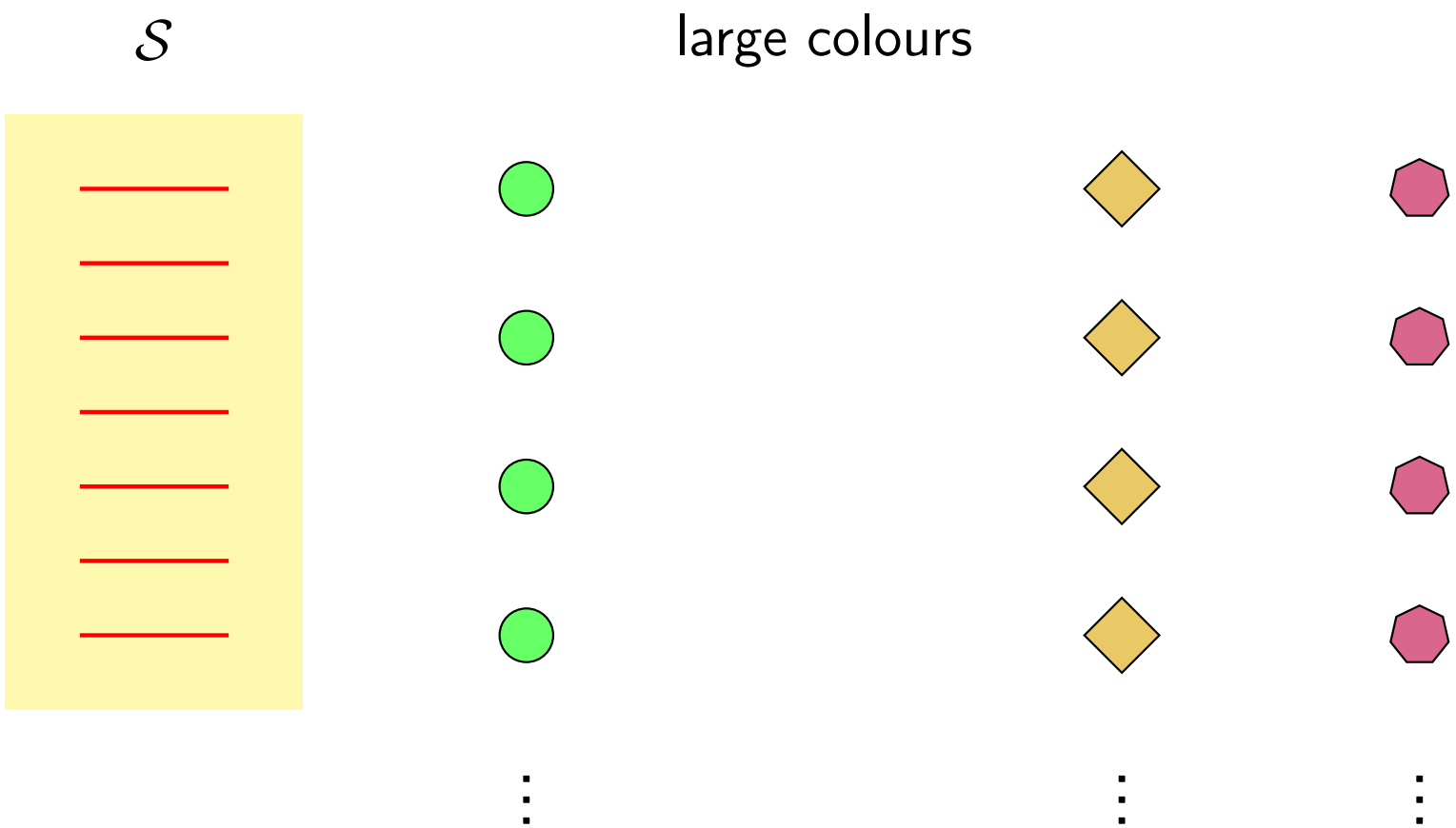

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Enumeration under updates $f(\varphi, d) = 3\text{-exp}(\|\varphi\| + \lg\lg d)$

Theorem (Berkholz, Keppeler, S. 2017): There is a dynamic algorithm that receives as input

- a degree bound $d \geqslant 2$,
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For the generalisation to FOC($\mathbb{P}$) (FO + counting + numerical predicates from $\mathbb{P}$) use a corresponding HNF for that logic [Kuske, S. 2017].

Positive Results for FO-Queries (static setting: without updates)

Constant delay enumeration after pseudo-linear time preprocessing is possible for all FO-queries φ on db-class C, if

- C has bounded degree [Durand, Grandjean 2007]
- C has bounded tree-width [Bagan 2006]
- C has bounded expansion [Kazana, Segoufin 2013]
- C has locally bounded expansion [Segoufin, Vigny 2017]
- C is nowhere dense [S., Segoufin, Vigny 2018]

  For any subgraph-closed class C that is not nowhere dense, evaluation of Boolean FO-queries is not possible in time $f(\|\varphi\|)\|D\|^{O(1)}$
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Open: To which extensions of FO can this be generalised?
Which results can be extended to the dynamic setting?

Overview

Constant delay enumeration after linear time preprocessing

- Let φ be a fixed query
- Let C be a class of databases
- Goal:
  Find an algorithm which, upon input of a db D,
  uses preprocessing time linear in $\|D\|$ and then
  enumerates $\varphi(D)$ with delay $O(1)$

- Question:
  For which φ, C is this possible?

Now: φ is a CQ $\qquad \varphi(x_1, \ldots, x_k) \;=\; \exists y_1 \cdots \exists y_\ell \big(R_1(\ldots) \wedge \cdots \wedge R_s(\ldots)\big)$

C is the class of all dbs

Obvious condition: Boolean φ must be tractable in linear time

$\rightsquigarrow$ restrict attention to acyclic CQs [Yannakakis 1981]

Examples (static setting for acyclic CQs)

$$\varphi_{S\text{-}E\text{-}T}(x,y) \quad := \quad S(x) \wedge E(x,y) \wedge T(y)$$

can be enumerated with constant delay after linear time preprocessing:

- For input db $D = (\mathrm{adom}(D), E^D, S^D, T^D)$
- Compute $E_1 := \{(x,y) \in E^D : x \in S^D\}$
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otherwise, Boolean Matrix Multiplication (for two $n \times n$ matrices A and B) could be solved in time $O(n^2)$, contradicting an algorithmic assumption. $n = |\mathrm{adom}(D)|$

Characterisation for the static setting

Definition: A CQ φ is acyclic if there exists a join tree, i.e., a tree t whose nodes are exactly the atoms of φ, and where for any two nodes u, v the following is true for every common variable y of u and v: y occurs in every node on the path between u and v in t.

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If φ is self-join free and not free-connex, then it is not possible to enumerate $\varphi(D)$ with constant delay after linear preprocessing, unless Boolean Matrix Multiplication can be done in time $O(n^2)$.

Dynamic setting

Queries that are easy in the static setting
can be difficult for the dynamic setting!

Example: $\varphi_{E\text{-}T}(x) \;:=\; \exists y \,\big(\, E(x,y) \,\wedge\, T(y) \,\big)\,.$

Why? — See the next few slides:

Introduce the notion of q-hierarchical CQs, which captures the easy CQs in the dynamic setting

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Dalvi & Suciu (2007) introduced the hierarchical CQs to characterise the Boolean CQs that can be answered in PTIME on probabilistic dbs.

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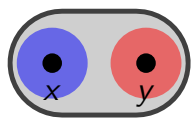

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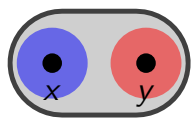

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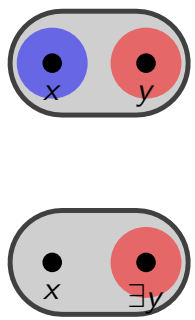

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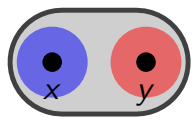

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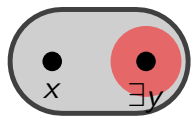

A q-hierarchical query:

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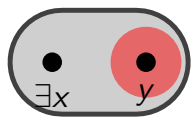

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A lower bound for enumerating via OMv

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This solves OMv in total time $O(n^{3-\varepsilon})$ ↯

Intractability result for Boolean CQs that are not q-hierarchical

The OuMv-problem: [Henzinger et al. 2015]

Input: a Boolean $n \times n$-matrix M and
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Fix an $\epsilon > 0$ and let φ be a Boolean CQ whose homomorphic core is not q-hierarchical.
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Proof idea for $\psi_{S\text{-}E\text{-}T} := \exists x \exists y \left(S(x) \wedge E(x,y) \wedge T(y) \right)$

Intractability result for counting CQs that are not q-hierarchical

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Input: two sets U and V of n Boolean vectors of dimension $d := \lceil \log^2 n \rceil$

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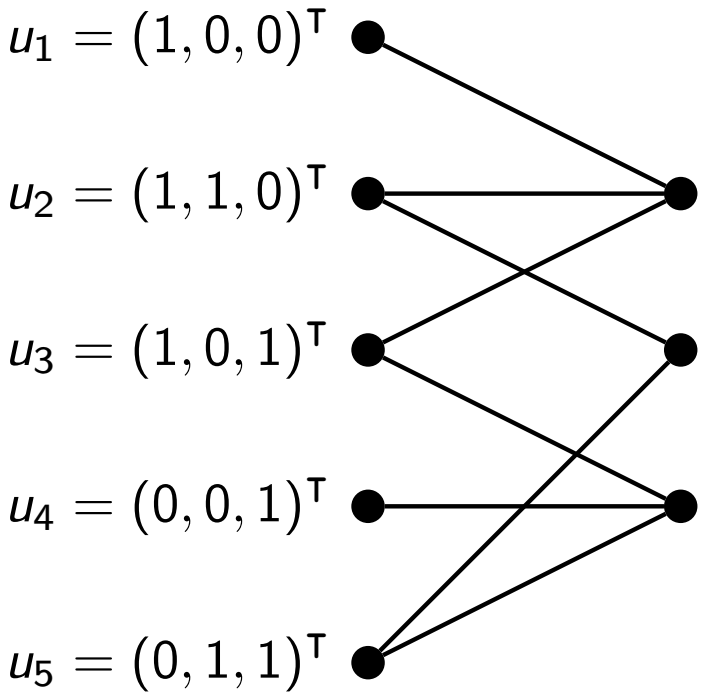
A lower bound for counting via OV

Left: n vertices for the n vectors $u \in U$
Right: $d := \lceil \log^2 n \rceil$ vertices for vector-coordinates



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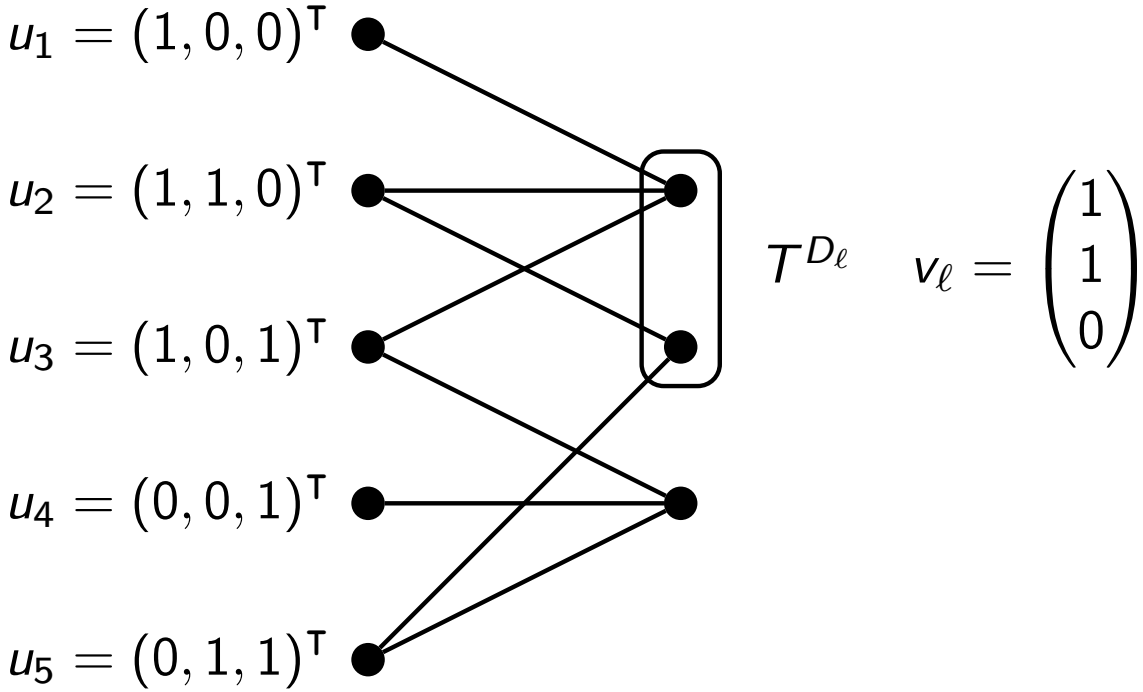

- for each $v_\ell \in V$: update T^{D_ℓ} in time $d \cdot n^{1-\epsilon} = \lceil \log^2 n \rceil n^{1-\varepsilon}$

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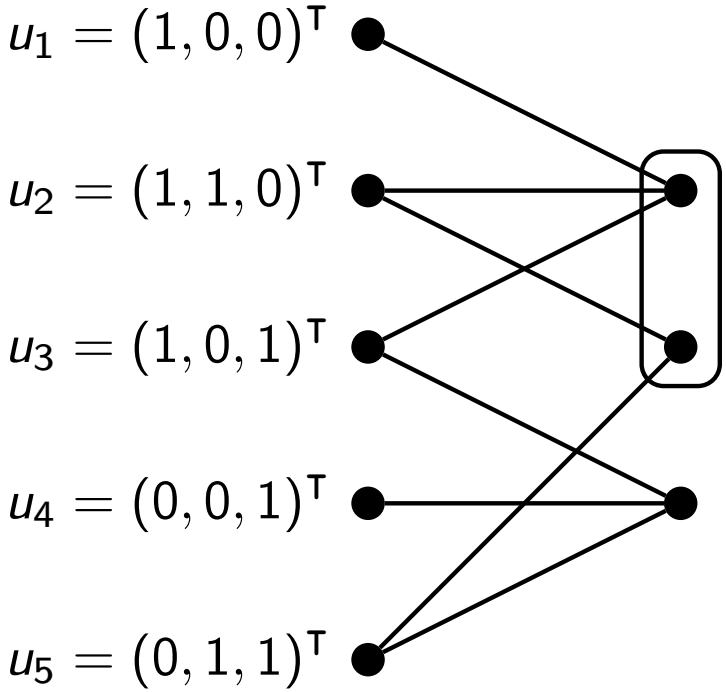

$T^{D_\ell} \quad v_\ell = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

$\big|\varphi_{E\text{-}T}(D_\ell)\big| = 4 = \big|\{\, u_i \;:\; {u_i}^\mathsf{T} v_\ell \neq 0 \,\}\big|$

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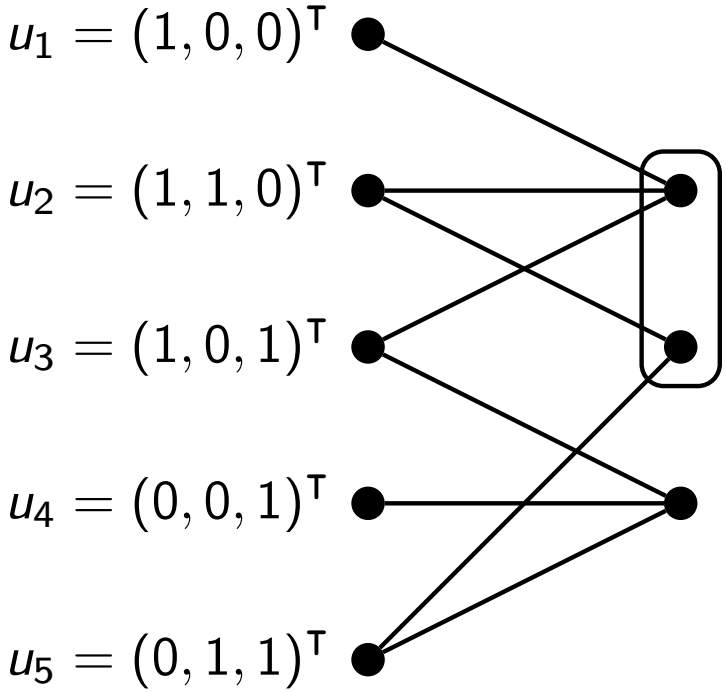

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- finished for all $v_\ell \in V$ within time $n \cdot \lceil \log^2 n \rceil n^{1-\varepsilon} = n^{2-\varepsilon'}$ ↯

Efficient evaluation of a fragment of CQs

Theorem (Upper bound) [Berkholz, Keppeler, S. 2017]:

For every CQ that is q-hierarchical, there is a dynamic data structure that has constant update time and allows to

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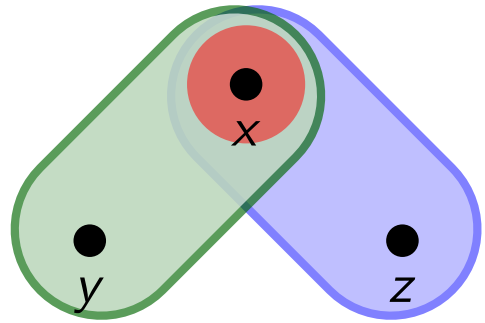
For every CQ that is q-hierarchical, there is a dynamic data structure that has constant update time and allows to

- answer a Boolean CQ,
- count the number of result tuples,
- enumerate the result relation with constant delay.

q-hierarchical queries

$$\varphi(x, y, z) \;:=\; R(x) \wedge E(x, y) \wedge F(x, z)$$



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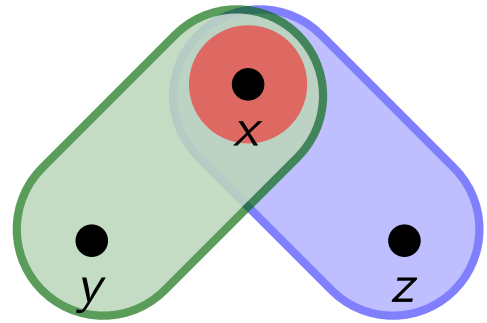

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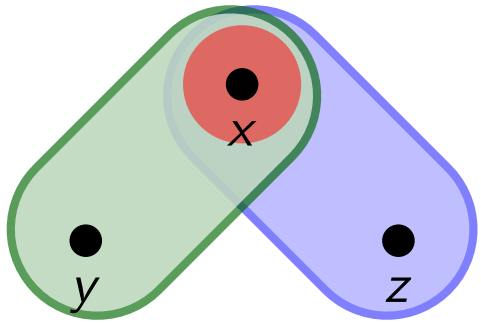

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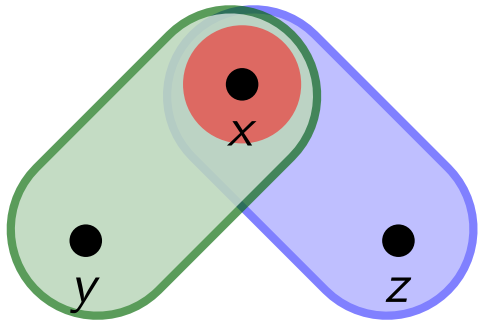

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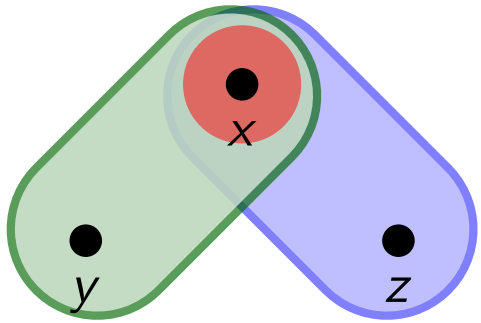

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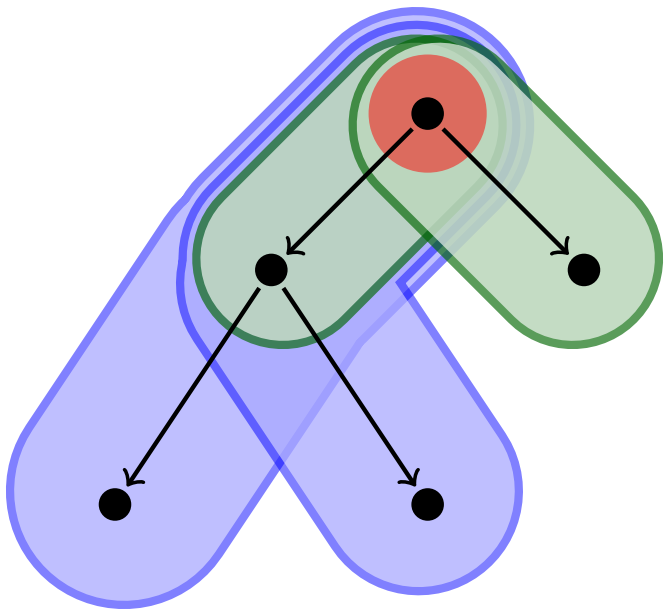

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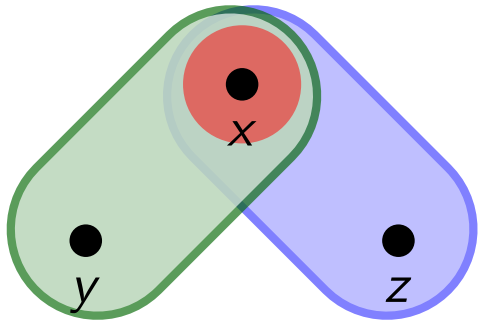

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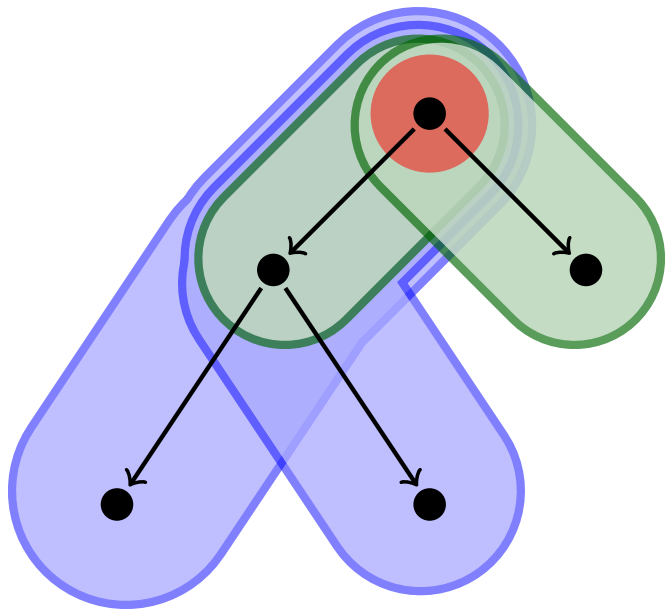

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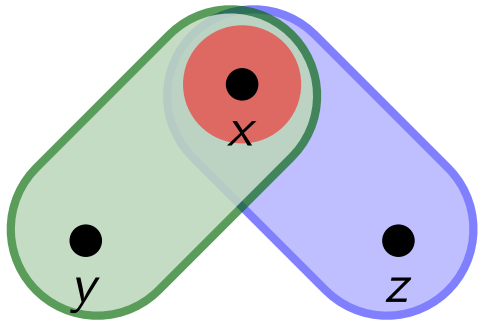

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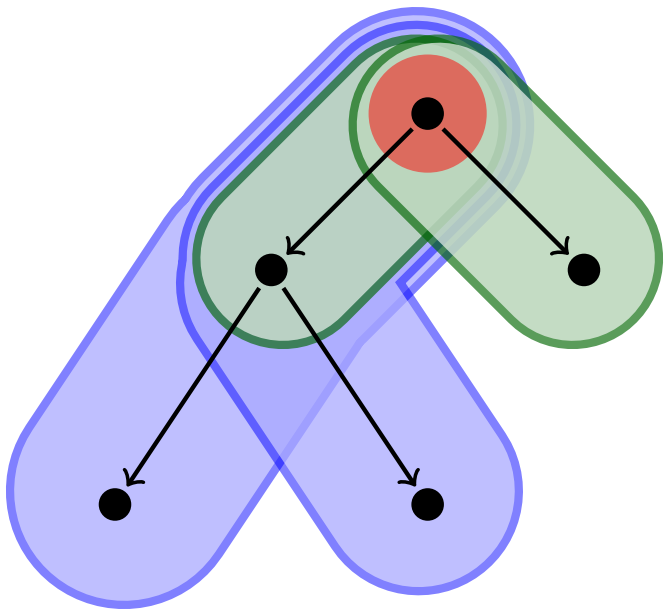

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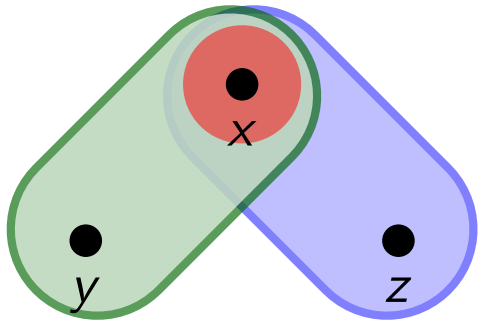

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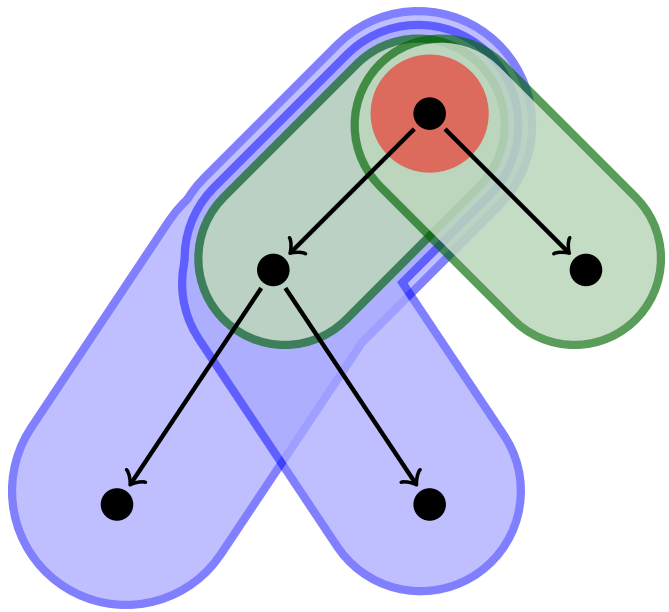

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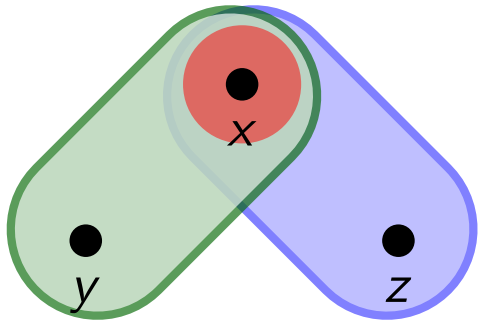

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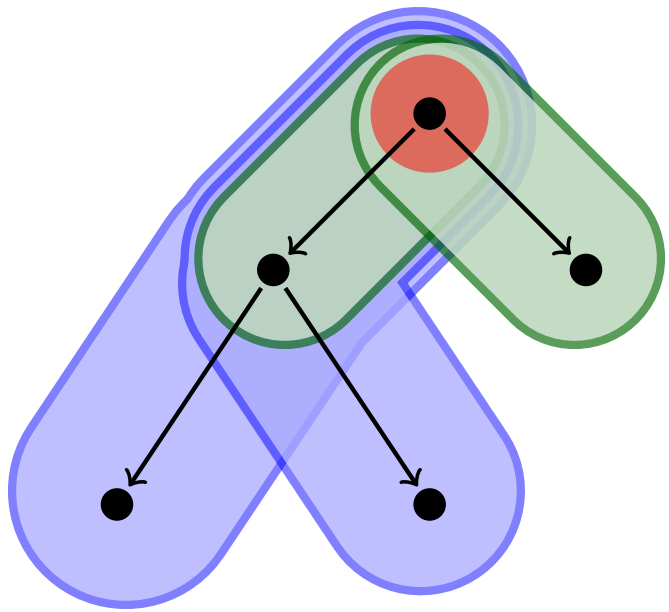
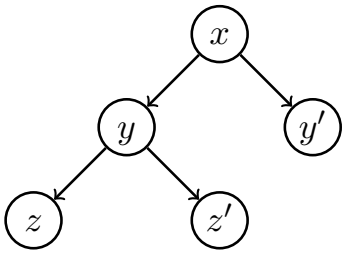
2. the free variables $\{x_1, \ldots, x_\ell\}$ form a connected subtree that contains the root



Lemma: A CQ $\varphi(\overline{x})$ is q-hierarchical $\iff$ every connected component of $\varphi(\overline{x})$ has a q-tree.

Data structure for q-hierarchical queries

$$\varphi(x, y, z, y', z') = \big(Rxyz \wedge Rxyz' \wedge Exy \wedge Exy' \wedge Sxyz \big)$$



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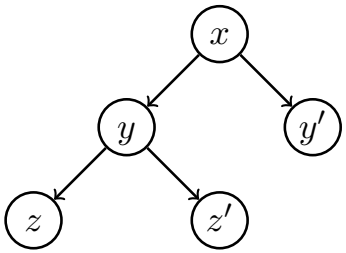

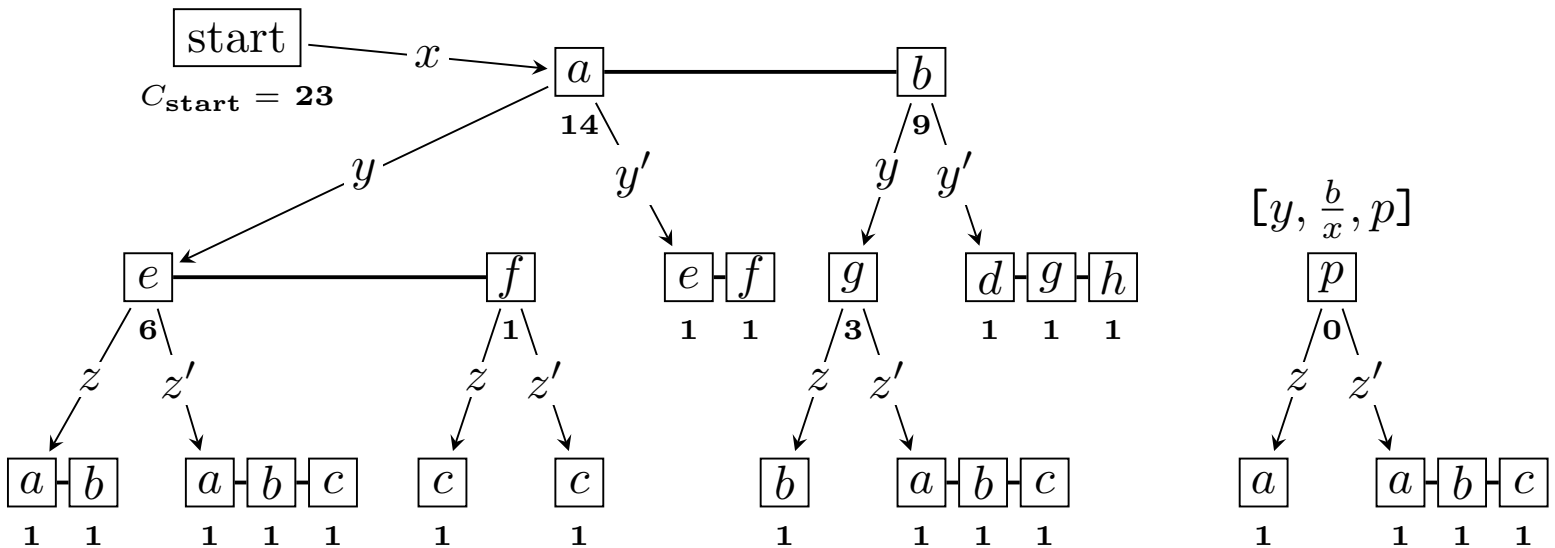

- $S(b, p, a), R(b, p, a), R(b, p, b), R(b, p, c) \in D,\ E(b, p) \notin D$

Data structure for q-hierarchical queries

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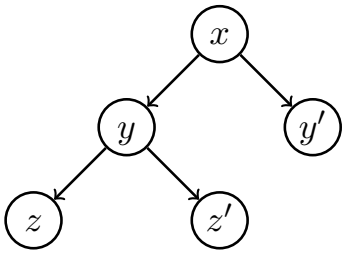

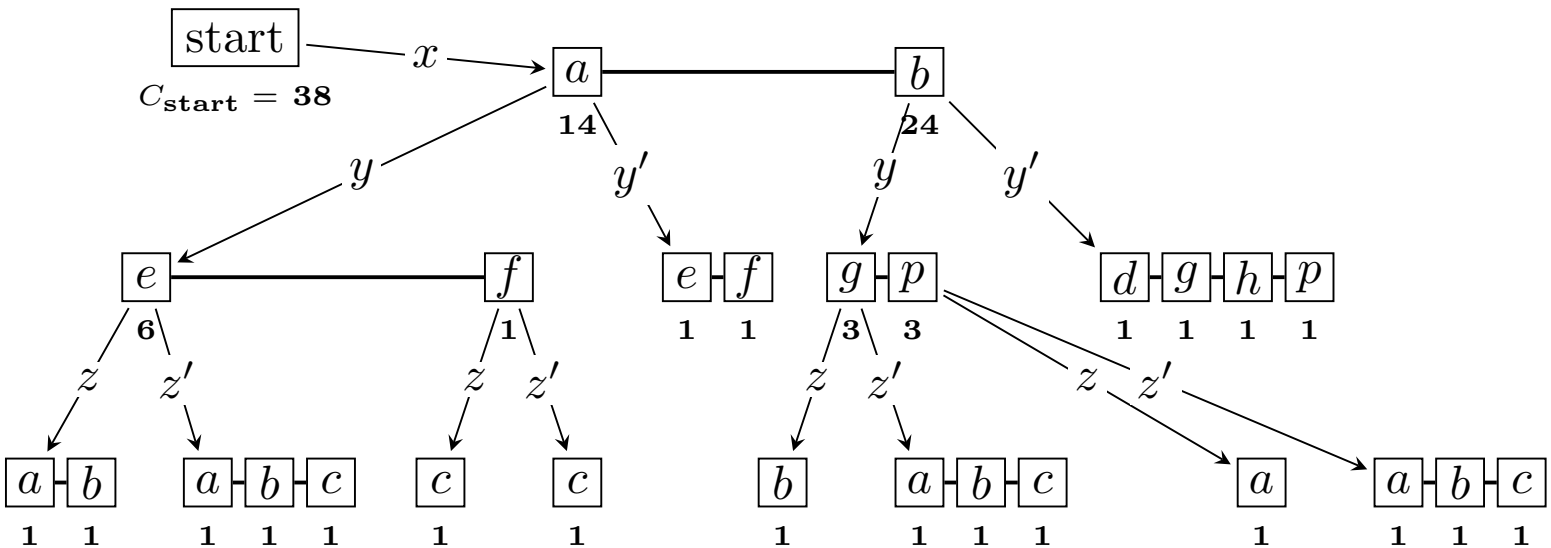

- $S(b, p, a), R(b, p, a), R(b, p, b), R(b, p, c) \in D,\ E(b, p) \notin D$
- INSERT $E(b, p)$

Overview

Positive Results for FO-Queries (static setting: without updates)

Constant delay enumeration after pseudo-linear time preprocessing is possible for all FO-queries φ on db-class C, if

- C has bounded degree [Durand, Grandjean 2007]
- C has bounded tree-width [Bagan 2006]
- C has bounded expansion [Kazana, Segoufin 2013]
- C has locally bounded expansion [Segoufin, Vigny 2017]
- C is nowhere dense [S., Segoufin, Vigny 2018]

  For any subgraph-closed class C that is not nowhere dense, evaluation of Boolean FO-queries is not possible in time $f(\|\varphi\|)\|D\|^{O(1)}$ (assuming FPT $\neq$ W[1]) [Kreutzer 2011, Dvořák, Král, Thomas 2010]

- C has low degree [Durand, S., Segoufin 2014]

  i.e., for each $\delta > 0$, all sufficiently large D in C have degree $\leqslant \|D\|^{\delta}$.

  Some such C are not nowhere dense (and not subgraph-closed).

Open: To which extensions of FO can this be generalised?
Which results can be extended to the dynamic setting?

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– Thank you for your attention! –